Multi-Branched Blood Vessels Segmentation Based on Phase-Field and Statistical Model

Shifeng Zhao, Mingquan Zhou, Zhongke Wu, Yun Tian, and Lizhi Xie

***Abstract*—The precise segmentation of cerebral vessels is essential for the detection of cerebrovascular diseases. The complex structures of cerebral vessels and the low contranst of thin vessels in medical images make precise segmentation difficult. In this study, we propose a new phase-field and statistical model for blood vessel segmentation. The proposed model is based on the Allen-Chan equation with double well potential and statistical distribution function. The brain tissues in the image are modeled by Gaussian distribution while cerebral vessels are modeled by uniform distribution respectively. The region distribution information combined with the phase-field model is used in curve evolution. And the level set method is developed to implement the curve evolution to assure high efficiency of the cerebrovascular segmentation. Comparisons with the LBF model and LCV model show that our model can achieve better results with fewer iteration number and less time.**

***Index Terms*—Segmentation, cerebral blood vessel, intensity inhomogeneity, statistical model.**

I. Introduction

Vascular segmentation is one of the fundamental tasks in the diagnosis and treatment planning of many different pathologies including arteriovenous malformations (AVMs), aneurysm, thrombosis and cardiac disease. Thus, accurate detection and segmentation of brain blood vessels is of major importance to the radiologists [1], especially those dim small ones. Therefore the accurate method of performing blood vessel segmentation on medical images of different modalities is a subject of much research attention [2].

A variety of methods have been developed for segmenting blood vessels. From the technical aspect, there is no general technique that may be effectively applied to all modalities. They have to be highly adapted to the application in order to achieve good performance. One of the general approaches for image segmentation is the minimizer of the piecewise constant Mumford-Shah functional using the variational-PDEs. Mumford-Shan function model [3] was firstly proposed as a general image segmentation model. Using this model the image is decomposed into some regions. Inside each region, the original image is approximated by a smooth function. Since the great success of curve evolution [4] and level-set method [5] in image segmentation for Mumford-Shah type model, different approaches have been tried to apply such method to image segmentation [6]-[12]. For example, Chan and Vese(CV) [6] proposed a level-set framework to minimize the so-called piecewise constant model by assuming that an image consists of statistically homogeneous regions. But CV model has intrinsic limitations such as the unsuccessful segmentation of images with intensity inhomogeneity, the sensitivity to the placement of initial contour and the extraordinary time-consumption. So many methods have been put forward to solve the limitations of CV models [13]-[16]. Solem et al. [13] and xia et al. [14], they improved the initialization prcess, while in [15] their purpose is to reduce the computational load of the curve evolution. Wang *et al.* [16] introduced a local Chan-Vese model which utilized both global information and local image for segmentation. They also introduced a new penalizing energy and a new termination criterion to deal with the iteration process.

Recently, the Allen-Cahn equation [17], also known phase field model, has attracted the attention of some scholars, and it has also proved efficiency in image segmentation problems [18]-[23]. For example, Jung *et al.* [21] proposed a phase-field method to solve mutliphase piecewise constant segmentation problem. The method is based on the phase transition model of Modica and Mortola with a sinusoidal potential and a fitting term. It is a variational partial differential equation approach that is closely connected to the Mumford-Shah model. Chen [22] extended the sine-sinc model to Gaussian-distribution-like image. They chose a normalization of the original image as initialization of the iterations to help convergency and replace the sinc function by the exponential function to improve the efficiency of the model. Li [23] proposed a phase-field model which was based on the Allen-Cahn equation with a multiple well potential and a data-fitting term. By using the polynomial potential, they can derive a more efficient and accurate numerical scheme based on an operator splitting technique.

A major difficulty in segmentation of blood vessels is the intensity inhomogeneity. Though many methods are proposed to solve the problem of non-uniform characteristics, they are still build on the original MS model. And the fundamental nature of the MS model is piecewise smooth. So such method is still sensitive to the selection of initial curves or sensitive to noises. Meanwhile, non-uniform property is little taken into account during the process. Furthermore, most of them are tested on small size images. For larger ones, they may not be bound to get good segmenting results. Due to the disturbance of the noise and the volume effect in medical images, the intensity of different brain tissues overlap with each other. Also different imaging devices, and even the different imaging method of the same imaging device, the

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sensitivity and specificity of the different vascular are also different. So, it is unreasonable to take the original image as a piecewise constant model. As a matter of fact, in medial images, the intensity of various elements or components are also different, even the same tissue, they may take on different intensity and obey some certain distribution. The objective of this study is to propose a new method for blood vessel segmentation using phase-field and statistical model which is based on the Allen-Cahn equation with a double well potential and a statistical data fitting term.

The reminder of this paper is organized as follows: In Section II, LCV model for image segmentation is briefly reviewed. In Section III, the proposed model for image segmentation is given. Section VI is the implementation. In Section V, we exhibit experiment results for different kinds of images. We especially take a comparison our model with other two models. Finally, we close the paper with a short conclusion.

II. Description of the Proposed Model

A. Local Chan-Vese Model

The Local Chan-Vese(LCV) model proposed by Wang *et al*. [16] is composed of three terms,i.e.,global term,local term and the regularization term. And the energy functional can be described as :

$$E^{LCV}(c_1,c_2,C)=\alpha\cdot E^G+\beta\cdot E^L+E^R \quad (1)$$

where E^G , E^L , E^R represent the global term, the local term and the regularization term respectively.

The global term E^G is directly derived from the CV model:

$$\begin{aligned}E^G=&\int_{inside(C)}\left|f_0(x,y)-c_1\right|^2dxdy\\&+\int_{outside(C)}\left|f_0(x,y)-c_2\right|^2dxdy\end{aligned} \quad (2)$$

where f_0 is the given image, c_1 and c_2 are the intensity averages of f_0 inside C and outside C ,respectively.

The local term E^L using the local statistical information to improve the segmentation capability can be written as:

$$\begin{aligned}E^L=&\int_{inside(C)}\left|g_k*f_0(x,y)-f_0(x,y)-d_1\right|^2dxdy\\&+\int_{outside(C)}\left|g_k*f_0(x,y)-f_0(x,y)-c_2\right|^2dxdy\end{aligned} \quad (3)$$

where g_k is a averaging convolution operator with $k\times k$ window. d_1 and d_2 are the intensity average of difference image $g_k*f_0(x,y)-f_0(x,y)$ inside C and outside C ,respectively.

And the third term E^R includes length penalty term $L(C)$ relating to the length of the evoluting curve C and a metric term $P(\phi)$ which charactirizes how close a function ϕ is to a signed function:

$$E^R=L(C)+P(\phi)=\mu\cdot\int_C dp+\int_\Omega\frac{1}{2}\left(\left|\nabla\phi(x,y)\right|-1\right)^2dxdy \quad (4)$$

where μ is the parameter which can control the penalization effect of the length term.

B. New Model

Because the intensity of various brain tissues is not necessarily consistent. They may be subject to different distribution. We analyzed different images and found that the histogram always has one peak near the low intensity region, which is not surprising since intensity inhomogeneity often occurs in medical images. The distribution characteristics is meaningful feature which can facilitate segmentation.So in order to get better segmentation result, we extend the model in the following steps.

First, from the above analysis, we will assume that the image consists of two classes, vessel and non-vessel. Non-vessel class includes the low intensity region and is modeled by a Gaussian distribution. The vessel class includes arteries and is modeled by a uniform distribution. The distribution of the image can be expressed as a finite mixture of two classes: vessel and non-vessel:

$$f(x)=P(x\mid\omega_1)P(\omega_1)+P(x\mid\omega_2)P(\omega_2) \quad (5)$$

where $f(x)$ is the total distribution of image pixels. x is the intensity. $P(x\mid\omega_1)$ is the posterior distribution of non-vessel class. $P(x\mid\omega_2)$ is the posterior distribution of vessel class. $P(\omega_1)$ and $P(\omega_2)$ are the proportion of non-vessel and vessel in image respectively. A Gaussian distribution models non-vessel class as follows:

$$P(x\mid\omega_1)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad (6)$$

A uniform distribution models vessel class as follows:

$$P(x\mid\omega_2)=\frac{1}{I} \quad (7)$$

where, I is the maximum intensity in the image.

Second, inspired by the multiphase image segmentation via Modica-Mortola phase transition, we use Allen-Chan equation to replace the length of the segmenting the curve C and use the statistical distribution to replace the global term, then we will get get our phase-field and statistical model, so the energy functional can be written as:

$$\begin{aligned}E(\phi)=&\int_\Omega\left(\frac{F(\phi)}{\varepsilon^2}+\frac{|\nabla\phi|^2}{2}\right)dxdy+\\&\left[\int_{inside(C)}\left|g_k*f_0(x,y)-f_0(x,y)-d_1\right|^2dxdy+\right.\\&\left.\int_{outside(C)}\left|g_k*f_0(x,y)-f_0(x,y)-d_2\right|^2dxdy\right]+\\&\left(\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(f_0-\mu)^2}{2\sigma^2}\right)P(\phi)+\frac{1-P(\phi)}{I}\right)\sin c^2(\phi-k)\end{aligned} \quad (8)$$

where $F(\phi)=0.25(\phi^2-1)^2$ is a double well potential. The small constant ε is the gradient energy coefficient related to the interfacial energy, Ω is the image domain, $P(\phi)$ is the proportion of non-vessel class in the image. f_0 is the given image, μ is the mean, σ^2 is the variance of the Gaussian distribution, I is the maximum intensity in the image. And the constant functions d_1, d_2 are given as:

$$d_1=\frac{\int_\Omega\left[g_k*f_0(x,y)-f_0(x,y)\right]H_\varepsilon(\phi(x,y))dxdy}{\int_\Omega H_\varepsilon(\phi(x,y))dxdy} \tag{9}$$

$$d_2=\frac{\int_\Omega\left[g_k*f_0(x,y)-f_0(x,y)\right]\left[1-H_\varepsilon(\phi(x,y))\right]dxdy}{\int_\Omega\left(1-H_\varepsilon(\phi(x,y))\right)dxdy}$$

Once ϕ comes to a steady state, the evolving curve C will separates the blood vessels form the background. From this purpose, we seek a law of evolution in the form:

$$\phi_t=-gradE(\phi). \tag{10}$$

The symbol 'grad' here denotes the gradient in the space $L^2(\Omega)$.then we have:

$$\phi_t=-gradE(\phi)=-\frac{F'(\phi)}{\varepsilon^2}+\Delta\phi$$

$$+\left[\left(g_k*f_0(x,y)-f_0(x,y)-d_1\right)^2\right.$$

$$\left.+\left(g_k*f_0(x,y)-f_0(x,y)-d_2\right)^2\right]$$

$$-\lambda\left[\left(\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(f_0-\mu)^2}{2\sigma^2}\right)P(\phi)+\frac{1-P(\phi)}{I}\right)\right]$$

$$\left(\frac{\sin(2\pi(\phi-k))}{\pi(\phi-k)^2}-\frac{2\sin^2(\pi(\phi-k))}{\pi^2(\phi-k)^3}\right) \tag{11}$$

where N is the number of pixels of the image, x_i is the intensity of pixel i.

III. Implementation

In our numerical experiments, we normalize the given image f as $f_0=\frac{f-f_{\min}}{f_{\max}-f_{\min}}$,where $f_{\max}$ and $f_{\min}$ are the maximum and the minimum values of the given image, respectively. When the distribution is Gaussian, the parameters of the distribution can be derived from the following:

$$P^{(n)}(\omega_k\mid x_i)=\frac{P^{(n)}(x_i\mid\omega_k)P\left(\omega_k^{(n)}\right)}{\sum_{j=1}^{2}P^{(n)}(x_i\mid\omega_j)P\left(\omega_j^{(n)}\right)} \tag{12}$$

$$P^{(n+1)}(\omega_1)=\frac{1}{N}\sum_{i=1}^{N}P^{(n)}(\omega_1\mid x_i) \tag{13}$$

$$\mu_k^{(n+1)}=\frac{\sum_{i=1}^{N}P^{(n)}(\omega_i\mid x_i)x_i}{\sum_{i=1}^{N}P^{(n)}(\omega_i\mid x_i)} \tag{14}$$

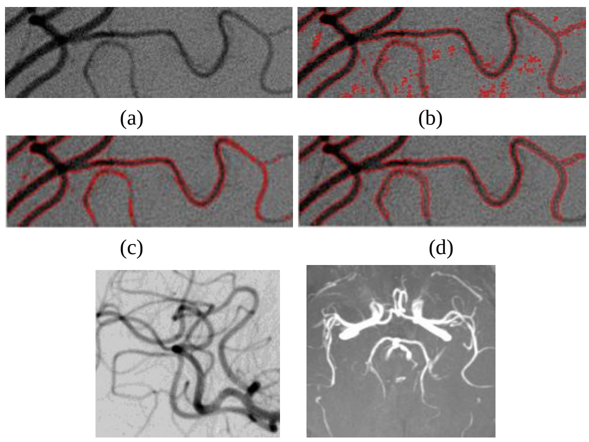
$$(\sigma^{(n+1)})^2=\frac{\sum_{i=1}^{N}P^{(n)}(\omega_1\mid x_i)\left(x_i-\mu^{(n+1)}\right)^2}{\sum_{i=1}^{N}P^{(n)}(\omega_1\mid x_i)} \tag{15}$$

IV. Expriment Results

In this section, the proposed algorithm has been tested on real blood vessels. Fig. 1 shows the segmentation results for three real blood vessel images with inhomogeneous intensity via use of the the Local Binary Fitting mode[12], Local Chan- Vese model[16] and the proposed method. In [16] they had compared the CV model with the LCV model. So we just compare LBF and LCV model with our proposed model. It can be seen from the results that the proposed model can achieve good segmenting results.

From the above experimental results, our method illustrates the ability of segmenting image with the intensity inhomogeneity and the images contain more perplexing vascular. The three images all have the characteristics of multi-branched. Because the real blood vessels in human brain take on such features as complicated and arranged in a crisscross pattern. As we all know,cerebral blood vessels are composed of a large number of branched structures,which form a highly entangled web. The methods used for blood vessel segmentation discussed in introduction can work well to deal with images of little size and the images just include one or two banch vessels. But actually, to deal with images containing more complicated branched blood vessels has more practical significance.

In real vessel images, within blood vessels, the intensity may be homogeneity, but surrongding the vicinity of vascular edges, the intensity are usually different. Because the background are normally composed of different brain tissues, they generaly have different density. So using different distribution functions to represent the different organizations is meaningful.



(a) (b)

(c) (d)

(e) (f)

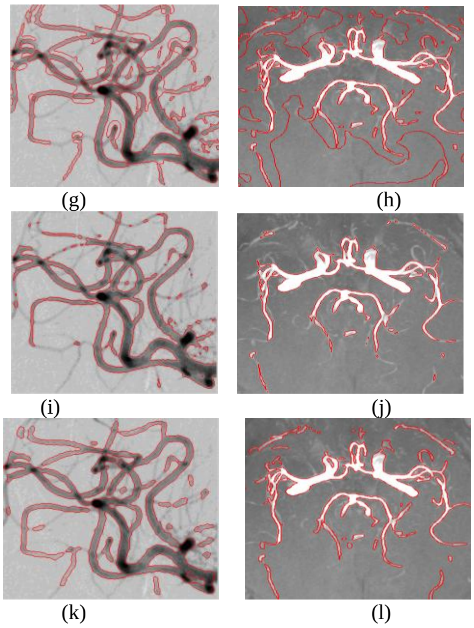


Fig. 1. The comparisons of the three models on segmenting real blood image with the intensity inhomogeneity . (a)(e)(f): original images. (b)(g)(h): final segmentation result using LBF model. (c)(i)(j): final segmentation result using LCV model. (d)(k)(l): final segmenting results using the proposed model.

Their iteration number and processing time for segmenting images in Fig. 1 are presented in Table I. It can be seen form the table that the iteration numbers and the processing time for the proposed model are both less than those of other two models.

For the table above, we also come to that, our proposed model not only has the ability of segmenting blood vessels with intensity inhomogeneity, but also has more advantages in handling larger images. When dealing with larger images, our method can get better results with less iteration number and less time.

TABLE I: THE PERFORMANCE COMPARISONS OF THREE MODELS

	size =255*77(a)		**size =319*294(e)**		**size =314*340(f)**	
	Iteration number	***Total time***	***Iteration number***	***Total time***	***Iteration number***	***Total time***
LBF model	20	16.7	50	52.2	100	88.9
LCV model	20	14.4	20	22.5 6	80	87.6
Proposed model	10	9.87	18	17.1	60	55.7

V. CONCLUSION

In this study, we propose a new model for blood vessel image segmentation, which is based on the techniques of curve evolution and global statistical distribution and level set theory. The energy functional for the proposed model consists of the double well potential and a statistical data fitting term. By incorporating the global statistical distribution information into the model, the images with intensity inhomogeneity can be efficiently segmented. The experiment results show that the proposed model is better than other similar methods in medical image segmentation. In our future work we will add more image properties and make use of different geometry features to improve the segmentation results and make our method more robust.

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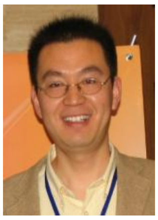

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